# Evolutionary Computation plus Dynamic Programming for the Bi-Objective Travelling Thief Problem 

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Project page: https://cs.adelaide.edu.au/~optlog/research/ttp.php Or google "travelling thief Adelaide"

Tuesday, July 17, 10:40-12:20, Conference Room D (3F)

## The Travelling Thief Problem (TTP)

Composed of the merging of the Traveling Salesman Problem and the Knapsack Problem


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## THE TRAVELING THIEF PROBLEM (TTP)

Goal: Visit each city exactly once, maximising the total profit $P$ such that the total weight does not exceed the knapsack capacity $W$, where $P$ is defined as:

$$
P=\sum_{i=1}^{m} p_{i} x_{i}-R \sum_{i=1}^{n} t_{i, i+1}
$$

where $x_{i}=\{1 \mid 0\}$ depending on whether the item $i$ is picked $\{1\}$ or not $\{0\}$, and $t_{i, j}$ is defined as:

$$
t_{i, j}=\frac{d\left(\Pi_{i}, \Pi_{j}\right)}{v_{\max }-W_{\Pi_{i}}\left(\frac{v_{\max }-v_{\min }}{W}\right)}
$$

where $\Pi_{i}$ is the city at tour position $i$ in tour $\Pi$, and $W_{\Pi_{i}}$ is the current weight of the knapsack at city $\Pi_{i}$.

## The Bi-Objective TTP

a natural extension: maximise the reward for a given weight of collected items, or determine the least weight subject to bounds imposed on the reward

- Objective one: profit P as defined before
- Objective two: total accumulated weight


## Packing-While-Travelling (PWT)

Definition 3.1. Let $\tau_{\pi}$ be a corresponding objective vector for $\bar{P}_{\pi}$. Then $\tau_{\pi}$ represents the related Pareto front designated as a $D P$ front for the given tour $\pi$.

(the "natural" approach would be the following)


## Solving the Bi-Obj. TTP

- Many single-objective TTP heuristics take a good TSP tour as a starting point. What does this mean here?
- TSP solvers; CONCORDE (CON), ACO, LKH and LKH2

eil76_n75_uncorr_01.ttp, inver over



## Algorithm 1 Hybrid IBEA Approach

Input: population size $\mu$; limit on the number of generations $\alpha$;
Initialisation:
set the iteration counter $c=0$;
populate $\bar{\Pi}$ with $\mu$ new tours produced by the TSP solver;
while $(c \leq \alpha)$ do
set $c=c+1$;

## Indicator:

run the DP for every tour $\pi \in \bar{\Pi}$ to compute its DP front $\tau_{\pi}$;
apply indicator function $\mathcal{I}\left(\tau_{\pi}\right)$ to calculate the indicator value for every individual tour $\pi \in \bar{\Pi}$;

## Survivor Selection:

repeatedly remove the individual with the smallest indicator value from the population $\bar{\Pi}$ until the population size is $\mu$ (ties are broken randomly);

## Parent Selection:

apply parent selection procedure to $\bar{\Pi}$ according to the indicator values to choose a set $\Lambda$ of $\lambda$ parent individuals;

## Mating:

apply crossover and mutation operators to the parents of $\Lambda$ to obtain a child population $\Lambda^{\prime}$;
set the new population as $\bar{\Pi}=\bar{\Pi} \cup \Lambda^{\prime}$;
end while

## Indicators

Def 3.2: Given $q$ different DP fronts, let $\phi$ denote a set of possible unique solution points derived by $\tau_{1} . . \tau_{g}$. Then $\omega$ is a Pareto front formed by the points of $\phi$ and $\underline{\omega}$ is named as the surface of $\phi$.

Given a tour $\tau_{\pi}$, and its corresponding solution set $T_{\pi}$ :

- Surface Contribution: number of objective vectors contributed by $T_{\pi}$
- Hypervolume: volume covered by $T_{\pi}$ w.r.t ( $0, \mathrm{C}$ )
- Loss of Contribution: $\quad L S C\left(\tau_{\pi}\right)=1-S C\left(\Phi \backslash T_{\pi}\right)$

$$
\operatorname{LHV}\left(\tau_{\pi}\right)=1-\frac{H V\left(\Phi \backslash T_{\pi}\right)}{H V(\Phi)}
$$

## Parent Selection Mechanisms

- Rank-Based Selection (RBS), Fitness-Proportionate Selection (FPS), Tournament Selection (TS), Arbitrary Selection (AS), Uniformly-at-Random Selection (UAR)

Crossover and Mutation Operators

- TSP-only: multi-point crossover, 2-opt mutation, jump


## Experimental Study

- 2 indicators X 8 parent selection strategies
- TTP instances from the classes eil51, eil76, eil101; three knapsack types

Assessment

- 30 repetitions, Welch's t-test with UAR as a baseline (like the Student's t-test, but more reliable when the two samples have unequal variances and unequal sample sizes)



# Comparison of bi-obj. approaches with singleobjective MA2B 

| MA2B |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| eil51_n50 |  | Mean | Max | SD |
|  | Uncorrelated | 2805.000 | 2855 | 27.814 |
|  | SimilarWeights | 1416.348 | 1460 | 47.906 |
| eil76_n75 | Bounded | 4057.652 | 4105 | 25.841 |
|  | Uncorrelated | 5275.067 | 5423 | 78.138 |
|  | SimilarWeights | 1398.867 | 1502 | 55.448 |
| eil101_n100 | Bounded | 3849.067 | 4109 | 139.742 |
|  | Uncorrelated | 3339.600 | 3789 | 388.360 |
|  | SimilarWeights | 2215.500 | 2483 | 235.905 |
|  | Bounded | 4949.000 | 5137 | 139.285 |
| FPS LHV |  |  |  |  |
| eil51_n50 | Uncorrelated SimilarWeights Bounded | Mean | Max | SD |
|  |  | 2828.728 | 2854.543 | 15.357 |
|  |  | 1413.044 | 1459.953 | 17.780 |
| eil76_n75 |  | 4229.149 | 4230.997 | 10.118 |
|  | Uncorrelated | 5445.624 | 5514.666 | 58.992 |
|  | SimilarWeights | 1477.680 | 1513.404 | 24.494 |
| eil101_n100 | Bounded | 4042.449 | 4108.760 | 38.805 |
|  | Uncorrelated | 3620.844 | 3943.425 | 222.815 |
|  | SimilarWeights | 2431.907 | 2482.462 | 52.265 |
|  | Bounded | 5094.246 | 5233.513 | 65.267 |
| FPS LSC |  |  |  |  |
| eil51_n50 | Uncorrelated SimilarWeights | Mean | Max | SD |
|  |  | 2810.509 | 2832.496 | 18.076 |
|  |  | 1426.135 | 1459.953 | 21.990 |
| eil76_n75 | Bounded | 4231.299 | 4241.199 | 1.881 |
|  | Uncorrelated | 5392.575 | 5514.666 | 73.029 |
|  | SimilarWeights | 1474.803 | 1513.404 | 21.346 |
| eil101_n100 | Bounded | 4054.815 | 4102.167 | 21.440 |
|  | Uncorrelated | 3664.369 | 3846.172 | 124.994 |
|  | SimilarWeights | 2436.374 | 2482.462 | 49.731 |
|  | Bounded | 5067.070 | 5233.513 | 55.587 |

MA2B by El Yafrani and Ahiod [GECCÓ16]

Fitness-Proportionate Selection Loss of Hypervolume Loss of Surface Contribution

## Summary

- Bi-Objective TTP: profit vs. weight
- Dynamic programming provides provably optimal trade-off fronts for a given tour
- Indicator-based EA with a population of tours: with "loss of surface contribution" and "loss of hypervolume"
- Best bi-objective approaches beat single-objective state-of-the-art

