A Modified Indicator-based Evolutionary Algorithm (mIBEA)

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Content

Motivation

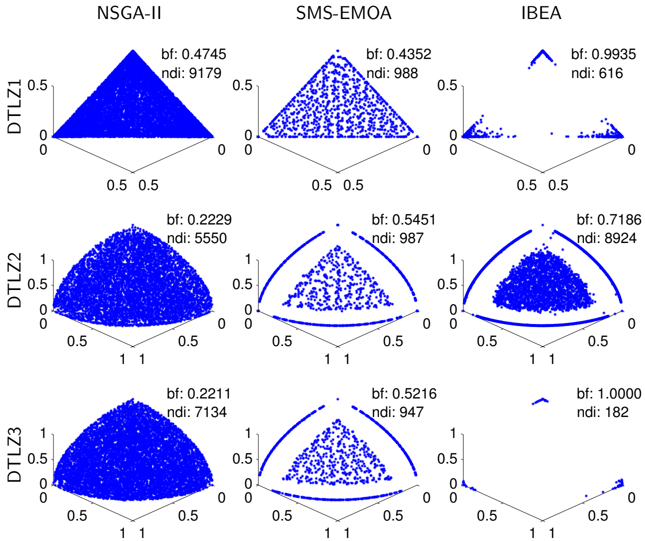

Figure: Non-dominated solutions from NSGA-II, SMS-EMOA and IBEA on DTLZ1-3 over 100 runs, except SMS-EMOA 10 runs due to high computation time.

Observations

ndi: # of non-dominated individuals.

border fraction (bf): $bf = \frac{ndi \ in \ A_{border}}{total \ ndi}$, $A_{border} := \{A | f_1 \leq \theta_1 \cup f_2 \leq \theta_2 \cup f_3 \leq \theta_3\}$.
$\theta_1 = \theta_2 = \theta_3 = 0.03$ for DTLZ1; $\theta_1 = \theta_2 = \theta_3 = 0.1$ for DTLZ2,3

Observations:

1. NSGA-II can cover the whole fronts of all DTLZ1-3;
2. SMS-EMOA reaches almost full coverage of the PF for DTLZ1, but leaves empty spaces on DTLZ2, 3;
3. IBEA heavily concentrates in corners on DTLZ1 and DTLZ3, while show clear gaps on DTLZ2;
4. $bf_{IBEA} > bf_{SMS-EMOA} > bf_{NSGA-II}$ on DTLZ1-3;
5. $ndi_{NSGA-II} \gg ndi_{IBEA}$ on DTLZ1 and 3;

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Question: why does IBEA deteriorate on DTLZ1 and 3 so severely?

Guess: Similar as the hypervolume drops of SMS-EMOA during the search [1]?

[1] Judt, L., Mersmann, O., Naujoks, B. (2013) Non-monotonicity of Observed Hypervolume in 1-Greedy S-Metric Selection. *Wiley Online Library*, 277–290.

IBEA VS *m*IBEA

*m*IBEA and IBEA difference: *m*IBEA removes dominated solutions before scaling objective values. No new parameters is introduced in *m*IBEA.

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Initialisation;
Randomly generate the initial population with population size μ;
while (!Stopping Criteria) do
    Remove dominated solutions using fast non-dominated sorting (NSGA-II);
      1) rank the solutions in P: Ranking rankedP = new Ranking(P);
      2) get the non-dominated solutions: P = rankedP.getSubfront(0);
    Objective values scaling ;
      1) find the lower (b_i = min_{x∈P} f_i(x)) and upper (b̄_i = max_{x∈P} f_i(x)) bound of each
     objective i.;
      2) increase the upper bound b̄_i' = b_i + ρ * (b̄_i − b_i);
      3) scale each objective to the interval [0 1]; f_i' = (f_i(x) − b_i)/(b̄_i' − b_i);
    Fitness assignment (hypervolume difference) using scaled objectives;
    Environmental Selection (Survival);
      1) remove the solutions with least hypervolume loss iteratively;
      2) update fitness values of all the remaining individuals;
    Mating Selection;
    Variation;
end
```

Algorithm 1: *m*IBEA Pseudo Code

Overall Results

With the same default settings as IBEA,

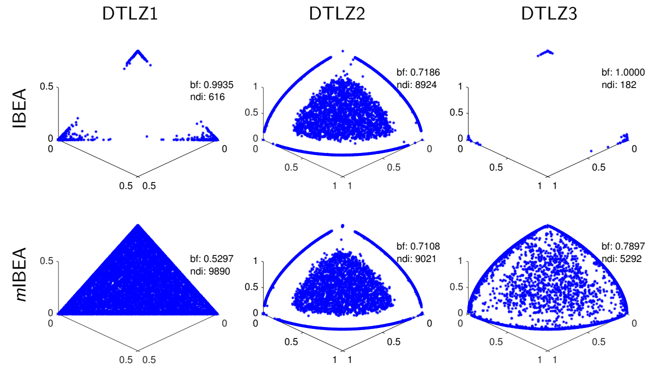

Figure: Non-dominated solutions from IBEA and *m*IBEA on DTLZ1-3.

*m*IBEA clearly improves the front coverage of IBEA on DTLZ1 and 3, but not on DTLZ2. (objective values evolving for DTLZ1: [100, 150]→[0,0.5], DTLZ2: [2, 5]→[0,1], DTLZ3: [500, 1500]→[0,1])

Performance Analysis w.r.t. different scaling factor ρ with population size $\mu = 100$

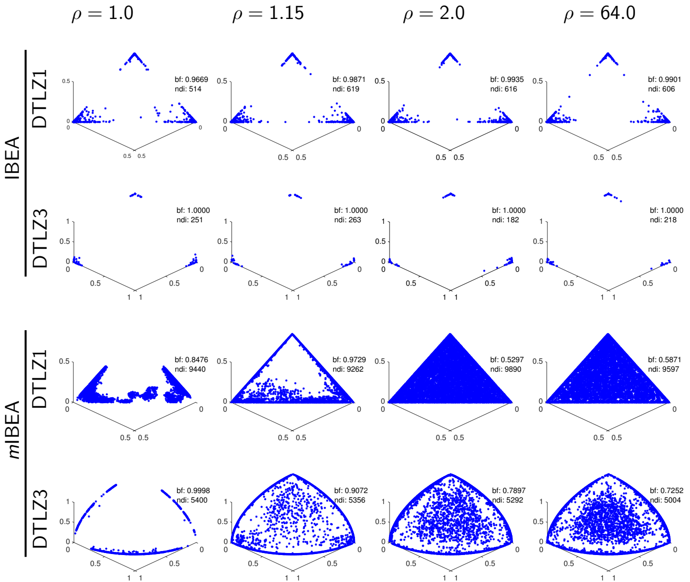

Figure: Non-dominated solutions of DTLZ1 and 3 from IBEA and *m*IBEA ($\mu = 100$).

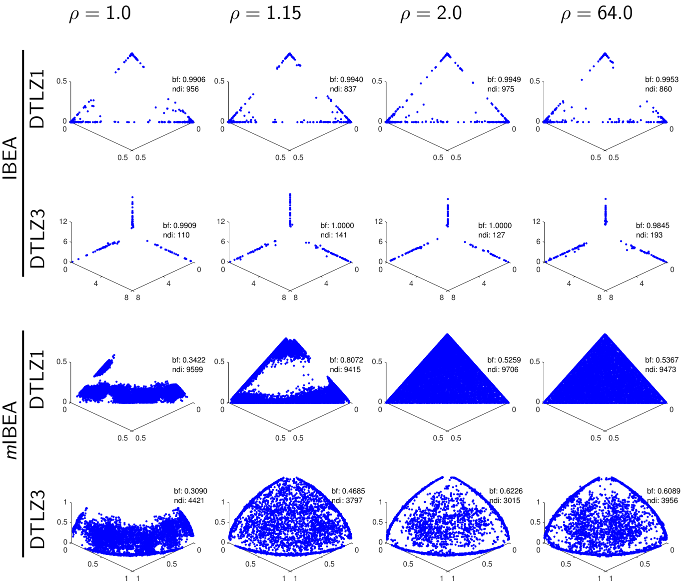

Figure: Non-dominated solutions of DTLZ1 and 3 from IBEA and *m*IBEA ($\mu = 1000$).

Performance Statistics and Running Time Comparison

Table: Performance Comparison

$\rho = 2.0$ $\mu = 100$	hypervolume IBEA	hypervolume *m*IBEA	$\epsilon+$ IBEA	$\epsilon+$ *m*IBEA
DTLZ1	$0.1773_{0.08}$	$\mathbf{0.7338_{0.10}}$	$0.2919_{0.02}$	$\mathbf{0.0771_{0.07}}$
DTLZ2	$0.4215_{0.00}$	$\mathbf{0.4217_{0.00}}$	$0.0724_{0.00}$	$0.0726_{0.00}$
DTLZ3	$0.0011_{0.01}$	$\mathbf{0.2623_{0.13}}$	$\mathbf{0.4217_{0.00}}$	$0.4514_{0.31}$
DTLZ4	$0.3023_{0.12}$	$0.2876_{0.14}$	$0.3596_{0.31}$	$0.3863_{0.34}$
DTLZ5	$0.0930_{0.00}$	$\mathbf{0.0931_{0.00}}$	$0.0129_{0.00}$	$\mathbf{0.0117_{0.00}}$
DTLZ6	$0.0789_{0.01}$	$\mathbf{0.0833_{0.01}}$	$0.0464_{0.02}$	$\mathbf{0.0271_{0.01}}$
DTLZ7	$0.2512_{0.05}$	$\mathbf{0.2693_{0.04}}$	$1.0914_{1.06}$	$\mathbf{0.7017_{0.77}}$

Table: Running Time Comparison

	Average Running Time per Run (in minutes)			
	$\mu = 100$		$\mu = 1000$	
$\rho = 2.0$	IBEA	*m*IBEA	IBEA	*m*IBEA
DTLZ1	0.15	**0.08**	7.86	**1.22**
DTLZ2	0.15	**0.10**	7.62	**1.46**
DTLZ3	0.15	**0.06**	8.10	**0.24**
DTLZ4	0.14	**0.11**	7.98	**1.56**
DTLZ5	0.16	**0.11**	8.01	**0.97**
DTLZ6	0.15	**0.11**	8.04	**0.21**
DTLZ7	0.16	**0.10**	8.03	**1.15**

Summary:

- ρ affects *m*IBEA significantly, but little impact on IBEA since all the solutions already collapse to the extreme points;
- μ has little influence on IBEA while solving DTLZ1 (as all the solutions are located in the border area), but deteriorates on DTLZ3. *m*IBEA improves with larger μ.
- *m*IBEA outperforms IBEA on both convergence (hypervolume, $\epsilon+$) and front coverage perspectives. However, *m*IBEA doesn't beat NSGA-II on most DTLZ problems.
- Over 8-fold speed-ups are obtained from *m*IBEA comparing with IBEA

Future Work

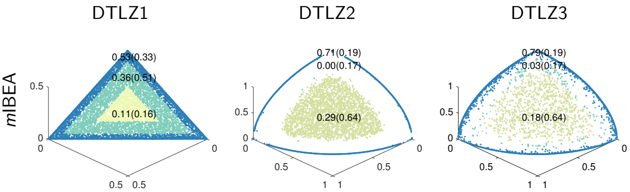

Figure: Solution distribution of *m*IBEA on DTLZ1-3

- *m*IBEA still have uneven distribution while solving DTLZ1 and gaps on DTLZ2 and 3 (see above figure).
- Systematically examine the courses of the gaps of IBEA's solution fronts on DTLZ2 and 3;
- Combining the investigation of hypervolume drops in SMS-EMOA to see if the gaps are caused by the computation of hypervolume;
- Improve *m*IBEA and SMS-EMOA by redirecting its attention away from extreme points.

Thank you!

Q & A