THEORETICAL RESULTS ON BET-AND-RUN AS AN INITIALISATION STRATEGY

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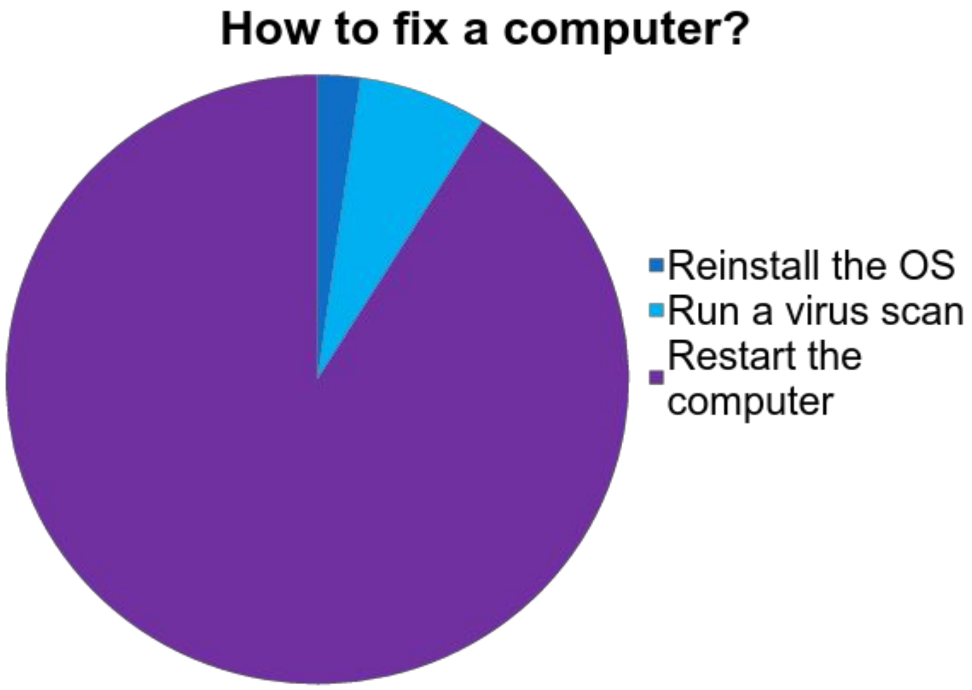
HOUSTON, WE HAVE A PROBLEM…

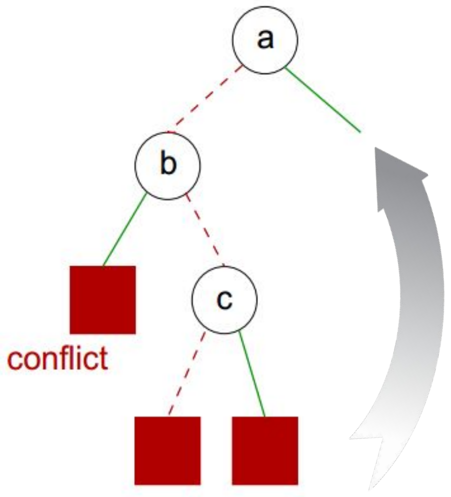
Restarts to the rescue!

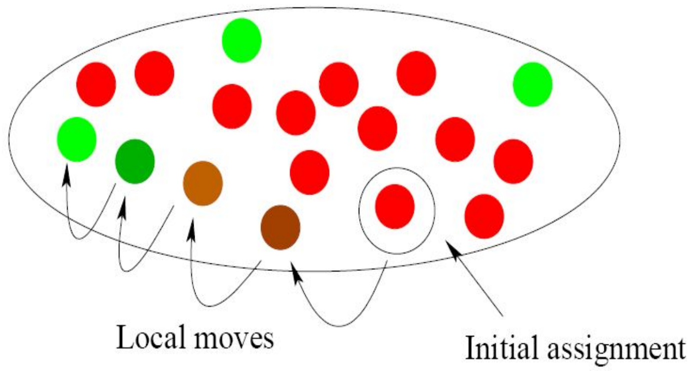


BACKGROUND

Restarted Search

- Become integral part of combinatorial search
- **Complete methods:** avoid heavy-tailed distribution (Gomes et al. JAR'00)
- **Incomplete methods:** diversification technique



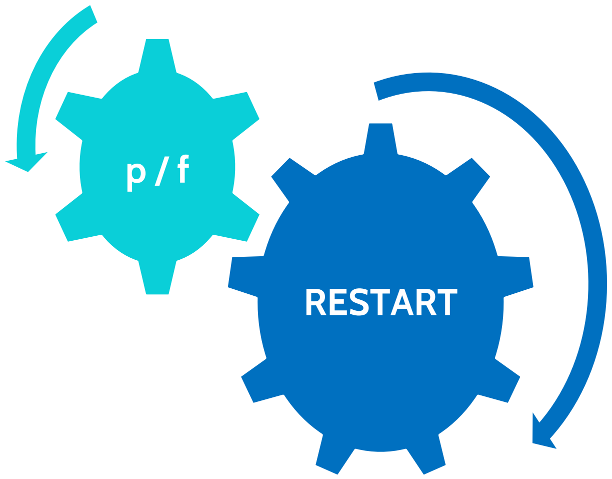


RESTARTS: BACKGROUND

BACKGROUND

Restart Strategies

- ➢ **Complexity** of designing appropriate restart strategy
- ➢ **Two common** approaches:
  1. Use restarts with a certain **probability**
  2. Employ **fixed** schedule of restarts



BACKGROUND

Restart Strategies – Feasibility

- ➢ Theoretical work on fixed-schedule restart strategies (Luby et al.'93)
- ➢ Practical studies with SAT and CP solvers
- ➢ Geometrically growing restarts limits (Wu et al. CP'07)
- ➢ (Audemard et al. CP'12) argued fixed schedules are sub-optimal for SAT

Restart Strategies – Optimization

- ➢ Classical optimization algorithms are often deterministic
  As such, does not really benefit from restarts
- ➢ Modern optimization algorithms have randomized components
  Memory constraints & parallel computation introduce new characteristics
- ➢ (Ruiz et al.'16) different mathematical programming formulations to provide different starting points for the solver

LIMITED RUNTIME BUDGET

Restart Strategies

➢ Assume we are given a **time budget *t*** to run an algorithm

LIMITED RUNTIME BUDGET

Restart Strategies

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- Two natural options:
  1. **Single–run strategy:** use all of the time *t* for a single run of the algorithm
  2. **Multi–run strategy:** make *k* runs each with runtime *t/k*

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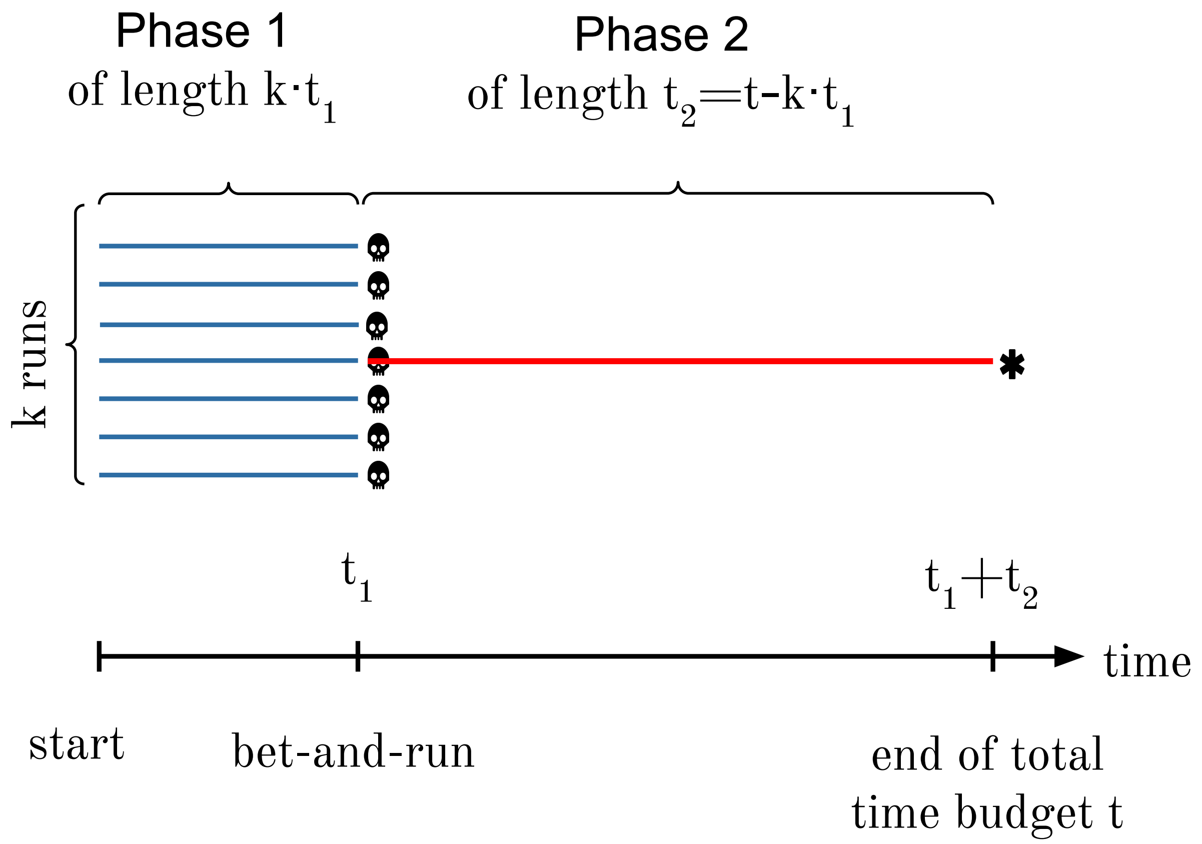
- (Fischetti et al.'14) generalizes this strategy into **Bet–And–Run** for MIPs

LIMITED RUNTIME BUDGET
BET-AND-RUN BY FISCHETTI AND MONACI (2014)



Another way to interpret this:
degenerated island model, without migration, and the greedy removal of islands

BET–AND–RUN: recent related work

Sampling Phase + Long Run

- ➢ (Fischetti et al. OR'14) introduced diversity in starting conditions of MIP Experimentally good results with $k = 5$
- ➢ (de Perthuis de Laillevault et al. GECCO'15) analysed 1+1-EA on OneMax, t_1=*1step*. A small additive runtime gain, hardly noticeable in practice.
- ➢ (Friedrich, Kötzing, Wagner AAAI'17) studied TSP and MVC Experimentally good results with $\text{Restarts}_{1\%}^{40}$
- ➢ (Kadioglu, Sellmann, Wagner LION'17) learned reactive restart strategies that considers runtime features.
- ➢ (**Lissovoi, Sudholt, Wagner, Zarges GECCO'17**) theoretical results for a family of pseudo-boolean functions. **Summary: non-trivial k and t_1 are necessary** to find the global optimum efficiently.

THEORY

OUTLINE

We analyse the Bet-And-Run strategy:

- with Randomised Local Search (and in some cases a (1+1) EA)
- on a simple artificial benchmark function.

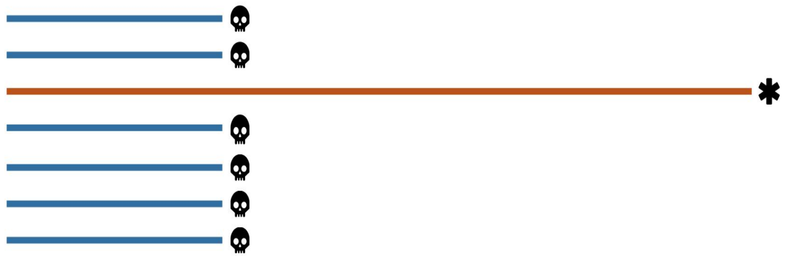
Aiming to answer:

- How does the algorithm behave with given k, t_1, t_2?
- Expected time to find the optimum?
- Expected fitness after $t = k \cdot t_1 + t_2$ iterations?
- How to choose t_1 and k?

BET-AND-RUN and RANDOMISED LOCAL SEARCH

Given a budget of $t = k \cdot t_1 + t_2$ fitness evaluations:

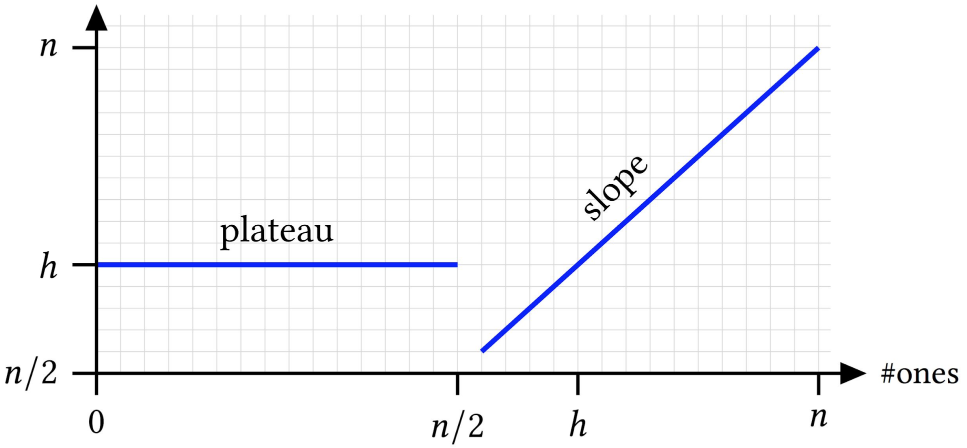
```
1. Run k instances of RLS independently for t_1 steps:
    a. Initialise a solution x uniformly at random.
    b. for i = 2 to t_1 do
        i. Let y be a mutation of x, flipping one bit chosen uniformly at random.
        ii. If f(y) ≥ f(x), replace x with y.
2. Choose run with highest fitness f(x).
3. Continue only this run for another t_2 steps.
```



PLATEAU / SLOPE FUNCTION FAMILY

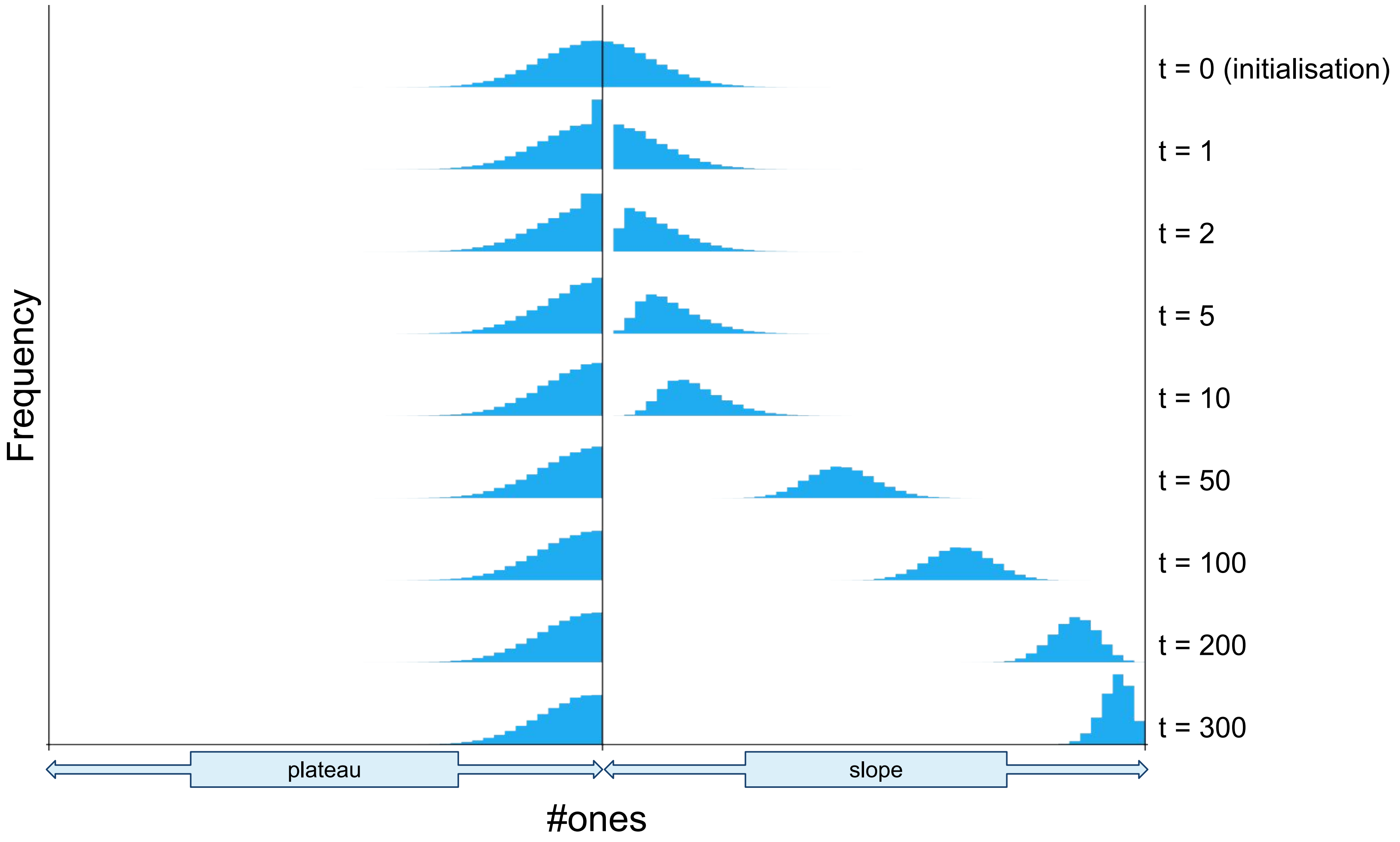
- Individuals are strings of n bits.
- Number of 1-bits affects fitness:
  - Plateau of fitness h when $|\mathbf{x}|_1 \leq n/2$
  - Slope when $|\mathbf{x}|_1 > n/2$
- Family characterised by $h > n/2$

- The plateau is easy to find...
  - ... and hard to escape from.
- The slope is initially worse...
  - ... but leads to the optimum.



$$f_h(x) = \begin{cases} |x|_1 & \text{if } |x|_1 > n/2 \\ h & \text{otherwise} \end{cases}$$

A SINGLE RUN OF RLS



INITIAL PHASE MUST BE LONG ENOUGH

When t_1 is large enough, an on-slope run will climb above the plateau.

Consider f_h with $h > n/2 + n^{0.5} \log n$. For any constant $\varepsilon > 0$,

- If $t_1 \geq (1+\varepsilon)\, n \ln(n/(2n - 2h))$, (and $k \geq c \log n$ for a constant $c > 0$,)
  With probability at least $1 - (3/4)^k - O(1/n)$, the optimum is found after $O(kn \log n)$ fitness evaluations.
- If $t_1 \leq (1-\varepsilon)\, n \ln(n/(2n - 2h))$, (and $k \leq \text{poly}(n)$,)
  With probability at least $1 - 2^{-k} - e^{-\Omega(\sqrt{n})}$, the optimum is **never** found.

The proof uses Fitness Levels with Tail Bounds (Witt '14).

FIXED BUDGET ANALYSIS OF A SINGLE RLS RUN

Where do we expect to be after t iterations?

- If initialised on the plateau, still on the plateau.
- If initialised "safely" on the slope, some distance up the slope.
  - Fixed budget analysis of RLS on OneMax (Jansen/Zarges '14) applies in this case.
- If initialised on the first point of the slope, split *almost* equally.
  - It is slightly easier to get to the plateau.

Combined, the expected fitness after t iterations of a single RLS run is:

- $E(f_h(x_t)) \geq n/2 + h/2 - (n/4 - 1) \cdot (1 - 1/n)^t$
- $E(f_h(x_t)) \leq n/2 + h/2 - (n/4 - 0.5\, n^{0.5} \log n) \cdot (1 - 1/n)^t + \Omega(n^{0.5})$

FIXED BUDGET FOR BET-AND-RUN

When **k** and **t_1** are sufficiently large, at least one run reaches $f_h(x_{t_1}) > h$ with high probability. We bound the expected fitness of the bet-and-run strategy using the fitness achieved by a slope run after t_1+t_2 iterations.

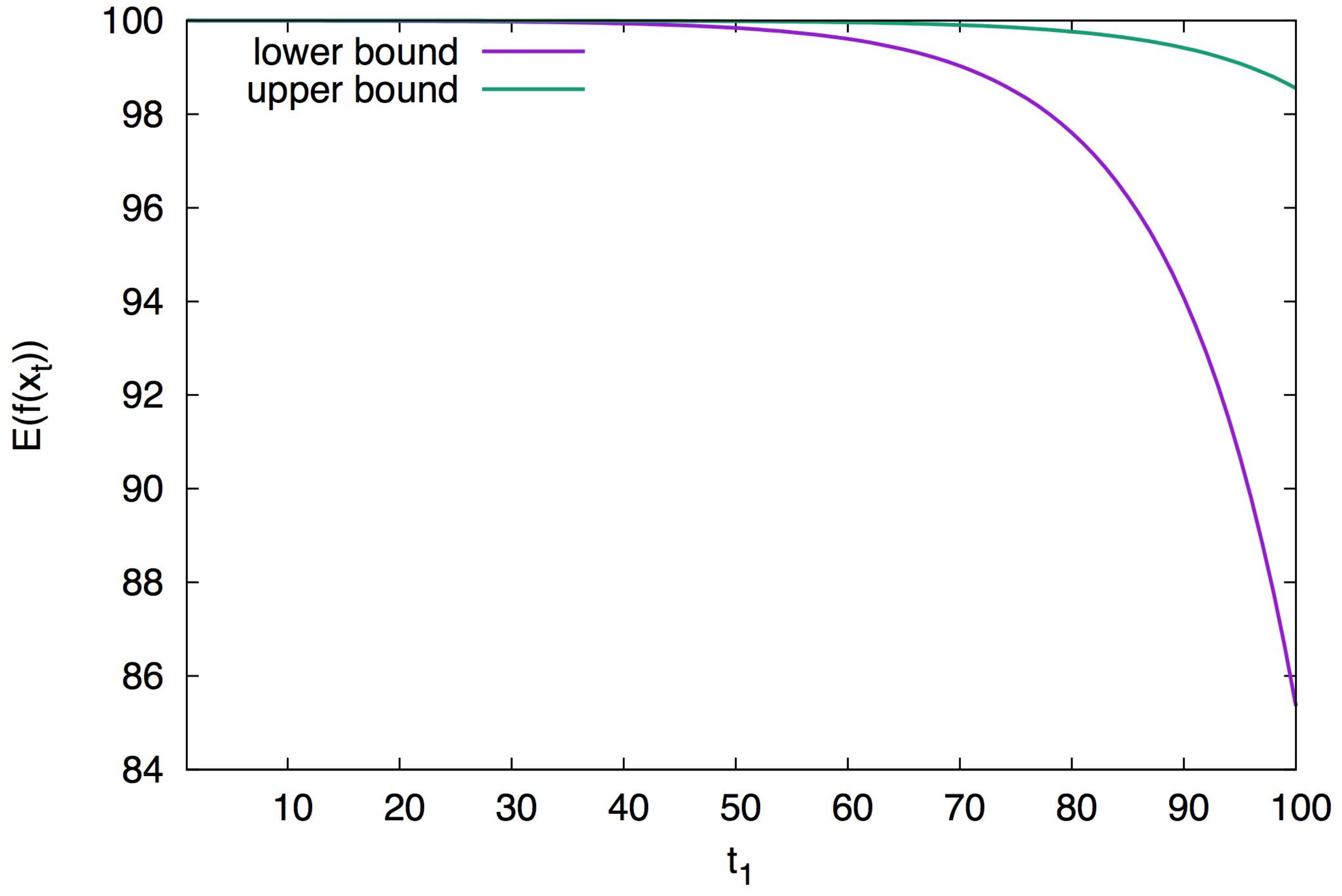
The expected fitness of RLS with a bet-and-run strategy, using $c \log n \leq k \leq \text{poly}(n)$ and $t_1 \geq (1+\varepsilon)n \ln(n/(2n-2h))$, after $t = k \cdot t_1 + t_2$ steps is:

- $E(f(x)) \geq n - (n/2 - d\, n^{0.5}) \cdot (1 - 1/n)^{t - (k-1)t_1} - (3/4)^k\, n$
- $E(f(x)) \leq (1+\square)\, (n - (n/2 - n^{0.5} \log n) \cdot (1 - 1/n)^{t - (k-1)t_1}) + o(1)$

for all $t \geq 0$, and d, $\square$, $\varepsilon > 0$ constant.

Consequence: should not set **t_1** or **k** excessively large.

EXCESSIVE T_1 IS DETRIMENTAL



SUMMARY

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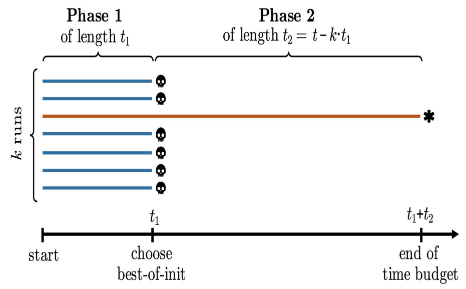
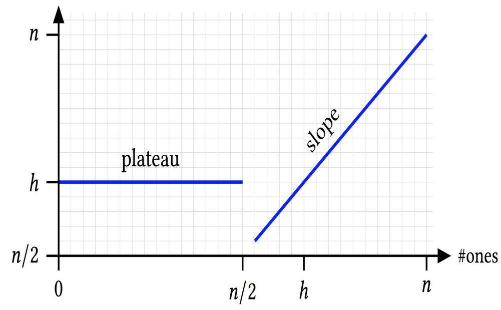
- Mathematically proven: bet-and-run can be an effective countermeasure when facing problems with deceptive regions.
- Complementary experiments are in the paper.

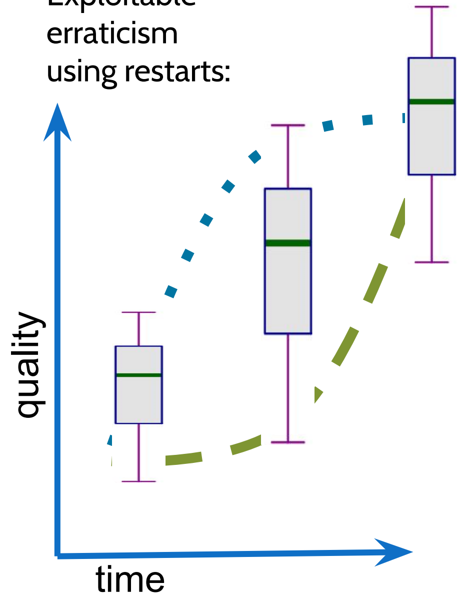




Future work

- Multi-modal functions
- Characterise progress variance of runs in Phase 1 so that this can be exploited in theory and practise.

Exploitable erraticism using restarts:



Special Issue on

„Algorithm Selection and Configuration in Evolutionary Computation“

— Submission Deadline: November 30, 2017 —

JOURNAL:

- Evolutionary Computation Journal
- MIT Press (http://ecj.napier.ac.uk)

POSSIBLE TOPICS (not limited to those):

- automated algorithm selection
- specific machine learning concepts
- configuration methods
- performance analysis
- features and diversity problem instances
- benchmarking concepts
- exploratory landscape analysis

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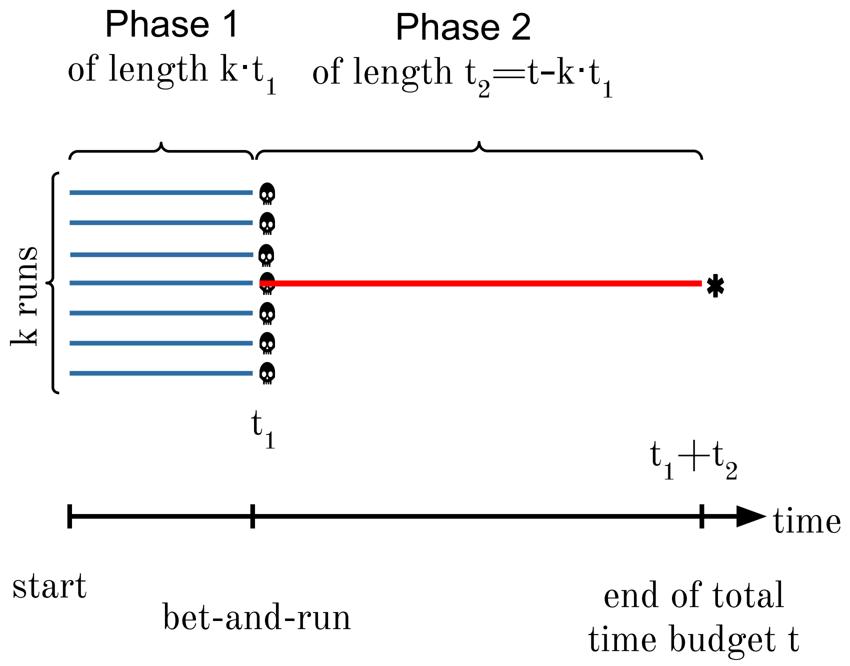

Notes
Single-run:
  k=1
Multi-run with restarts from scratch:
  $t_1=t/k$ and $t_2=0$

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degenerated island model, without migration, and the greedy removal of islands