# A fitness landscape analysis of the Travelling Thief Problem


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## ABSTRACT

Local Optima Networks are models proposed to understand the structure and properties of combinatorial landscapes. The fitness landscape is explored as a graph whose nodes represent the local optima (or basins of attraction) and edges represent the connectivity between them. In this paper, we use this representation to study a combinatorial optimisation problem, with two interdepend components, named the Travelling Thief Problem (TTP). The objective is to understand the search space structure of the TTP using basic local search heuristics and to distinguish the most impactful problem features. We create a large set of enumerable TTP instances and generate a Local Optima Network for each instance using two hill climbing variants. Two problem features are investigated, namely the knapsack capacity and profit-weight correlation. Our insights can be useful not only to design landscape-aware local search heuristics, but also to better understand what makes the TTP challenging for specific heuristics.



## KEYWORDS

Fitness Landscape, Basins of attraction, Local Optima Networks, Multi-component problems, Travelling Thief Problem

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## 1 MOTIVATION

In combinatorial optimisation, a fitness landscape of a given problem can be defined as a graph where nodes represent solutions, and edges represent the existance of a neighbourhood relation given a move operator [27]. However, there are two main issues with this representation. Firstly, defining the neighbourhood matrix for the entire set of possible solutions can be a very expensive task even for small instances. Secondly, it is hard to extract useful information about the search landscape given the fitness landscape graph.

Ochoa et al. [17] proposed a simplified landscape representation called Local Optima Networks (LONs). In this representation, the fitness landscape is represented as a graph whose nodes are associated with local optima (or basins of attraction) and edges indicate connectivities between the local optima. Two basins of attraction are connected if at least one solution within a basin has a neighbour solution within the other given a defined move operator. LONs provide a very useful representation of the search space of combinatorial problems, which can be exploited mainly by using measures and indices from the graph theory. LON characteristics have also been found to correlate with the performance of heuristic search algorithms [12].

In this paper, we use this representation to study the Travelling Thief Problem (TTP), a combinatorial optimisation problem that was originally introduced by Bonyadi et al. [2] and reformulated by Polyakovskiy et al. [19]. The TTP was designed to reflect some important aspects found in real-world optimisation problems such as the composite structure and the existence of interdependencies. The problem received considerable attention particularly from the evolutionary computation community. Designing efficient heuristic solvers is the focus of most of works on TTP [16, 25, 30], only few studies have been conducted to understand the problem complexity [20, 26, 29].

Wu et al. [29] investigated how the renting rate parameter impacts the difficulty of TTP instances, and proposed intervals (lower and upper bounds) to increase the instances' hardness. The authors exploited the obtained results to implement an instance generator able to create hard-to-solve instances for simple evolutionary algorithms. Note that two other problem parameters that also impact the problem difficulty are the number of cities and the number of items per city. In this paper, we study two non-trivial problem properties, namely the knapsack capacity and the correlation between the weights and the profits of the items.

Our main goal is to understand the search space structure of the TTP using basic local search heuristics, and to distinguish the most impactful non-trivial problem features. Thus, we study the knapsack capacity and the profit-weight correlation. In this work, we (i) propose a problem classification based on these two features, (ii) create a large set of enumerable TTP instances according to the features, (iii) generate a LON for each instance using two hill climbing variants, (iv) explore/exploit, using specific measures, the obtained LONs to gain some insights into their structure and characteristics[1].

The problem difficulty can be recognised using LON topological properties or information about the basins of attraction. Our analysis on such information corroborates the general idea that, using basic local search techniques there is a direct correlation between lower knapsack capacity and higher problem difficulty. In addition, other LON examinations revealed some real-world networks' characteristics such as cliqueness and sparse interconnectivity. We believe that these insights might contribute to the design of efficient local search-based heuristics for the TTP, and to understanding the search space structure of complex problems with interdependent components in general.

In the next section, we present the mathematical formulation of the problem and briefly review the fitness landscape topic including some state-of-the-art works. In Section 3, we describe the basic local search framework that will be used in our study. In Section 4, we propose a problem classification and briefly introduce the instance generator we used. The results are reported and analysed in Section 5. Finally, Section 6 concludes the paper.

# 2 BACKGROUND AND RELATED WORKS

## 2.1 The Travelling Thief Problem

The Travelling Thief Problem is a combinatorial optimisation problem that aims to provide testbeds for solving problems with multiple interdependent components [2, 19]. The TTP combines two well-known problems, namely, the Travelling Salesman Problem and the Knapsack Problem.

The problem is defined as follows: we are given a set of $n$ cities, the associated matrix of distances $d_{ij}$, and a set of $m$ items distributed among these cities. Each item $k$ is defined by a profit $p_k$ and a weight $w_k$. A thief must visit all the cities exactly once, stealing some items on the road, and return to the starting city.

The knapsack has a capacity limit of $W$, i.e. the total weight of the collected items must not exceed $W$. In addition, we consider a renting rate $R$ that the thief must pay at the end of the travel, and the maximum and minimum velocities denoted $v_{max}$ and $v_{min}$ respectively. Furthermore, each item is available in only one city, and $A_i \in \{1, \ldots, n\}$ denotes the availability vector. $A_i$ contains the reference to the city that contains the item $i$.

A TTP solution is coded in two parts: the tour $X = (x_1, \ldots, x_n)$, a vector containing the ordered list of cities, and the picking plan $Z = (z_1, \ldots, z_m)$, a binary vector representing the states of items (1 for packed, and 0 for unpacked).

To make the sub-problems mutually dependent, the TTP was designed such as the speed of the thief changes according to the knapsack weight. To achieve this, the thief's velocity at city $c$ is defined in Equation 1.

$$v_x = v_{max} - C \times w_x \tag{1}$$

where $C = \frac{v_{max} - v_{min}}{W}$ is a constant value, and $w_x$ the weight of the knapsack at city $x$.

We note $g(z)$ the total items value (defined in Equation 2), and we note $f(x, z)$ the total travel time (defined in Equation 3).

$$g(Z) = \sum_m p_m \times z_m \text{ S.T.} \sum_m w_m \times z_m \leq W \tag{2}$$

$$f(X, Z) = \sum_{i=1}^{n-1} t_{x_i, x_{i+1}} + t_{x_n, x_1} \tag{3}$$

where $t_{x_i, x_{i+1}} = \frac{d_{x_i, x_{i+1}}}{v_{x_i}}$ is the travel time from $x_i$ to $x_{i+1}$.

The objective is to maximize the total travel gain function, as defined in Equation 4, by finding the best tour and picking plan.

$$F(X, Z) = g(Z) - R \times f(X, Z) \tag{4}$$

## 2.2 Fitness landscape analysis

Fitness landscapes illustrate the association between search and fitness space, often depicted as rugged surfaces with many local peaks of different heights flanked by valleys of different depths [13, 14]. Given a specific landscape structure, defined as the triple $(S, F, N)$ where $S$ is the search space, $F$ the objective function, and $N$ the neighbourhood operator [27], a heuristic can be seen as a strategy for navigating this structure in the search for the highest peak or near-optimal solutions [22]. Fitness landscape analysis (FLA) has been applied to investigate the dynamics of evolutionary algorithms and single-solution heuristics for optimisation and design problems [24]. In addition, FLA can help predict the performance of those heuristics by using the search cost models for instance [22].

Landscape models have been used to make specific predictions regarding the behaviour of local search techniques, evolutionary algorithms, and other metaheuristics [28]. The behaviour is generally illustrated by the cost required to locate a solution with a given quality threshold given a problem instance. These models can also identify which features of the fitness landscape are responsible for the problem difficulty during the search process [27].

In order to understand the structural organisation of the local optima in combinatorial landscapes, Ochoa et al. [17] proposed the Local Optima Networks (LONs) as a simplified fitness landscape model. There, the fitness landscape is represented as a graph of connected local optima.

A local search heuristic $\mathcal{A}$ defines a mapping from the solution space $S$ to the set of locally optimal solutions $S^*$. A solution $i$ in the solution space $S$ is a local maximum given a neighbourhood operator $N$ if $F(i) \geq F(s), \forall s \in N(i)$. Each local optimum has an associated basin of attraction. In general, the basin of attraction of a local optima $i$ is the set composed of all the solutions that, after applying a local search procedure starting from each of them, the procedure returns $i$. Thereby, the basin of attraction associated to

[1] Our codes are publicly available at https://bitbucket.org/elkrari/ttp-fla

a local optimum $i$ is the set $B_i = \{s \in S | \mathcal{A}(s) = i\}$. The size of the basin of attraction of $i$ is the cardinality of $B_i$. Given a neighbourhood operator, a connection between two attraction basins is created if at least one solution in one basin has a neighbour solution in the other basin.

Figure 1 represents a simplified illustration of the attraction basins (black circles), their local optima (red big dots), the solutions that converge to the local optima when applying the local search (black small dots), and the connections between the local optima (blue lines). Note that the figure is kept simple for visualisation purposes, and more sophisticated heuristics with explorative operators are expected to result in many more interconnections between the attraction basins.

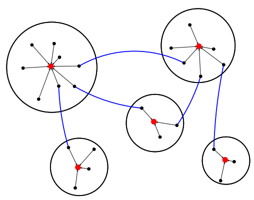


**Figure 1: A simplified illustration of the attraction basins and the connectivity in local optima networks.**

Local optima network properties for permutation-based problems have been studied in [5]. Furthermore, some works analysed the correlation between LON features and the performance of search heuristics [3, 18, 23].

In this paper two hill climbing local search procedures are investigated and the corresponding LONs are explored with the aim of understanding the difficulty of TTP instances. The LON model is adapted for the TTP. This way, we can understand the impact of some problem features by studying the topological structure of the generated LONs. The model is extended with additional measures that assess the connectivity in LONs, and statistical tests are used to explore the scale-freeness of the obtained LONs. Additionally, the attraction basins are also studied as they give additional insights into the difficulty of problem instances.

## 3 LOCAL SEARCH HEURISTICS

In this section, we present a basic local search framework for the TTP. The framework is a deterministic hill climber, designed with the only goal of investigating the structure of the problem search space. The pseudocode is described in Algorithm 1, and it can have multiple implementations depending on the chosen neighbourhood operators. Note that $\mathcal{N}_{TSP}(.)$ and $\mathcal{N}_{KP}(.)$ represent the neighbourhood functions for the TSP component and the KP components respectively.

**Algorithm 1** A basic local search heuristic framework for the TTP

```
s ← initial solution
while there is an improvement do
    for each s* N_TSP(s) do
        for each s** N_KP(s*) do
            if F(s**) > F(s) then
                s ← s**
            end if
        end for
    end for
end while
```

Although most state-of-the-art heuristics proposed for the TTP have a two-stage structure[2] [9, 16, 30], the use of an embedded neighbourhood structure is crucial for this study. In fact, two-stage heuristics divide the TTP solution into two components at each iteration and generate a large number of landscapes (one landscape for each sub-problem at each heuristic iteration) — this makes it virtually impossible to investigate the local optima structure of the overall problem. On the other hand, a joint-neighbourhood structure generates a problem specific neighbourhood function and preserves homogeneity of the TTP solutions. This helps the heuristic to easily and efficiently depict the structure and connectivity of local optima. This local search framework was explored by [8] and showed a competitive performance for small TTP instances compared with other basic stochastic heuristics. However, the approach shows some drawbacks, notably with scalability and exploration abilities.

In the context of this study, we consider two local search variants based on Algorithm 1. The first (named J2B) uses the *2-OPT* neighbourhood as the $\mathcal{N}_{TSP}(.)$ neighbourhood, while the second (named JIB) uses the *insertion* operator. In both variants, the one-bit-flip operator is used to construct the $\mathcal{N}_{KP}(.)$ neighbourhood.

## 4 EXPERIMENTAL SETTING

### 4.1 TTP classification and parameters

The TTP instances can be classified according to the following properties.

- **Number of cities ($n$):** The only parameter belonging to the TSP component in the TTP definition. The TTP benchmark library [19] gets its TSP component from the TSPLIB database which was introduced in [21].
- **Item Factor ($\mathcal{F}$):** Represents the number of items per city. Each city, except the first one, has the same number of items. The total number of items is $m = (n-1) \times \mathcal{F}$.
- **Profit-value correlation ($\mathcal{T}$):** Defines the correlation among the weight ($w_i$) and profit ($p_i$) of each item. Three correlations have been defined in the TTP library, namely, *uncorrelated (unc)*, *uncorrelated with similar weight (usw)*, and *bounded strongly correlated (bsc)*.
- **Knapsack capacity class ($C$):** Ranges between 2 and 10. $C$ is a factor occurring in the maximum weight of the knapsack which is given in Equation 5.

$$W = \frac{C}{11} \sum_{x=2}^{n} \sum_{y=1}^{\mathcal{F}} w_{xy} \tag{5}$$

The term $C/11$ is used to limit $W$, i.e., class $C = 10$ enlarges $W$ around to 90%, allowing more objects in the knapsack [19].

Since we are interested in studying the impact of non-trivial features on the structure of the fitness landscape, we classify the problem instances based on the features $\mathcal{T}$ and $C$.

[2]This family of heuristics solve the problems by tackling each sub-problem individually using a heuristic search. The process is then iterated multiple times depending on the stopping criteria.

### 4.2 Instance Generation

The number of all possible solutions for a TTP instance is at most $(n-1)! \times 2^m$, which makes the enumeration and study of the standard instances impractical. Thus, an instance generator has been specially implemented to produce enumerable instances. In order to generate the local optima networks and identify the basins of attractions, we use small instances with 7 cities and 6 items (one per city, except for the starting one).

The generator has been designed following the directives in [20]. The authors computed the renting rate $R$ (i.e., Equation 6) by using existing solvers to get the best picking plan for the KP[3] component and the near-optimal tour for the TSP.[4] As we are using small instances in this study, our generator obtains the renting rate for each instance by applying an exhaustive search to find the optimal solution for each component.

$$R = \frac{g(Z_{opt})}{f(X_{opt}, Z_{opt})} \tag{6}$$

where $Z_{opt}$ and $X_{opt}$ represent the optimal picking plan and the optimal tour respectively

The TSP component is fixed, i.e., the set of coordinates is the same for all the generated instances. As we are interested in two problem features, namely the knapsack capacity and profit-weight correlation, the capacity class is varied between $C = 2$ and $C = 10$, and all three correlation variants are considered.

Note that, for very small TTP instances, the first capacity class ($C = 1$) can not be used with the *uncorrelated with similar weights* instances where the knapsack weights are ranged in $[10^3, 10^3 + 10]$. Indeed, following Equation 5, for $n = 7$ and $\mathcal{F} = 1$, the minimal value of $W$ is $\frac{7}{11} \times 10^3$ which is smaller than the value of any items.

Therefore, 27 classes of the TTP are considered. For each class, 100 samples are generated with the aim of analysing their fitness landscapes.

## 5 RESULTS AND ANALYSIS

In this section, we analyse the local optima networks obtained using local search heuristics for the Travelling Thief Problem to achieve some insights about the structure of the search space. Furthermore, we study the basins of attraction and their relationship with some LON properties looking for additional information about the search difficulty.

### 5.1 Topological properties of local optima networks

Tables 1 and 2 report the average values for various network properties measured on TTP instances for the two local search heuristics. These properties are often used for LON analyses Ochoa et al. [17]. Values are averaged over 100 randomly generated instances, and subscript numbers represent the standard deviation. Each instance is represented by its corresponding LON.

The graph metrics are explained as follows. $\overline{n_v}$ and $\overline{n_e}$ represent the mean number of vertices (or nodes) and the mean number of edges over all the generated LONs respectively. $\overline{z}$ is the mean of the average degrees. $\overline{C}$ is the mean of the average clustering coefficients. $\overline{C_r}$ is the mean of the average clustering coefficients of corresponding random graphs (i.e. random graphs with the same number of vertices and mean degree). $\overline{l}$ is the mean of the average shortest path lengths between any two local optima. $\pi$ is the connectivity rate, which represents the probability over the 100 samples that the LON is a connected graph. Finally, $\overline{S}$ is the mean number of non-connected components (sub-graphs).

A first insight into the search difficulty can be drawn from the values of $\overline{n_v}$ and $\overline{n_e}$. We can clearly see that the number of vertices and the number of edges generally decrease when the knapsack capacities increase for both local search variants. This implies that the hardness of search decreases when the knapsack capacity increases. Intuitively, this makes sense, as instances with very large knapsack capacities are less constrained than those with medium and particularly small capacities.

The mean average degree $\overline{z}$ increases with the capacity class, and it decreases when the capacity class reaches 6 (7 in one case for JIB, $T = usw$).

We can also observe some so-called small-world properties by looking at the clustering coefficients ($\overline{C}$, $\overline{C_r}$) and the mean path lengths ($\overline{l}$). Firstly, the LONs show a significantly higher degree of local clustering compared with their corresponding random graphs. This means that the local optima are connected in two ways: dense local clusters and sparse interconnections, which can be difficult to find and exploit. Secondly, all the LONs have a small mean path length, i.e. any pair of local optima can be connected by traversing only few other local optima. A microscopic view on the values of the mean path length shows that it is proportional to $log(n_v)$. Therefore, a more sophisticated local search-based metaheuristics with exploration abilities, such as Tabu Search [10, 11], could move from a local optima to another within only a few iterations.

Interestingly, the connectivity rate shows that all the LONs generated using J2B are connected; while some of the LONs generated using JIB are disconnected graphs with a significantly high number of non-connected components, which is a notable disadvantage. This may not be surprising as the 2-opt and bit-flip operators, used in J2B, both induce a fully connected landscape, when considered separately for the KP and the TSP [27].

### 5.2 Degree Distributions

Figure 2 shows the cumulative degree distribution for some representative classes of the TTP for J2B and JIB in a log-log scale. The cumulative degree distribution function represents the probability $P(k)$ that a randomly chosen node has a degree larger than or equal to $k$. Though the figures show fluctuations, they allow us to deduce some interesting real-world network properties. We can see that the degree distributions decay slowly for small degrees, while their dropping rate is significantly faster for high degrees. This behaviour indicates that the majority of local optima have a small number of connections, while a few have a significantly higher number of connections.

Most of the real-world networks have their topological structure more accurately described by a power-law or a scale-free degree distribution $P(k) = k^{-\alpha}$, where $\alpha \in [2, 3]$ is a scaling parameter. The behaviour of local search strategies on networks has been studied according to the degree distribution [1]. Intuitively, the

[3] As available at http://www.diku.dk/ pisinger/codes.html
[4] Accessible at http://www.math.uwaterloo.ca/tsp/concorde/downloads/downloads.htm

**Table 1: General LON and basins' statistics for the J2B heuristic.**

| $\mathcal{T}$ | $C$ | $\overline{n_v}$ | $\overline{n_e}$ | $\overline{z}$ | $\overline{C_r}$ | $\overline{C}$ | $\overline{l}$ | $\pi$ | $\overline{S}$ |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | $412.57_{430.67}$ | $6820.46_{6059.62}$ | $35.57_{10.78}$ | $0.25_{0.25}$ | $0.64_{0.14}$ | $1.83_{0.33}$ | $1_0$ | $1_0$ |
| **unc** | 3 | $385.29_{405.53}$ | $8878.22_{9778.08}$ | $41.15_{17.19}$ | $0.25_{0.25}$ | $0.7_{0.14}$ | $1.47_{0.29}$ | $1_0$ | $1_0$ |
| | 4 | $269.09_{316.48}$ | $8816.11_{11170.62}$ | $47.55_{22.84}$ | $0.43_{0.31}$ | $0.77_{0.12}$ | $1.57_{0.32}$ | $1_0$ | $1_0$ |
| | 5 | $144.3_{205.49}$ | $4808.71_{8776.75}$ | $45.85_{23.36}$ | $0.59_{0.27}$ | $0.82_{0.1}$ | $1.4_{0.27}$ | $1_0$ | $1_0$ |
| | 6 | $98.69_{105.59}$ | $3435.64_{5583.68}$ | $47.54_{26.02}$ | $0.68_{0.22}$ | $0.86_{0.08}$ | $1.31_{0.22}$ | $1_0$ | $1_0$ |
| | 7 | $73.39_{122.2}$ | $2300.69_{6553.74}$ | $39.95_{21.72}$ | $0.78_{0.2}$ | $0.89_{0.08}$ | $1.21_{0.2}$ | $1_0$ | $1_0$ |
| | 8 | $36.75_{29.71}$ | $745.25_{1041.48}$ | $28.54_{16.86}$ | $0.88_{0.13}$ | $0.93_{0.05}$ | $1.11_{0.13}$ | $1_0$ | $1_0$ |
| | 9 | $37.81_{33.38}$ | $827.4_{1621.9}$ | $28.95_{18.61}$ | $0.88_{0.13}$ | $0.93_{0.05}$ | $1.11_{0.13}$ | $1_0$ | $1_0$ |
| | 10 | $29.84_{27.64}$ | $537.29_{979.27}$ | $23.42_{14.9}$ | $0.91_{0.12}$ | $0.94_{0.05}$ | $1.08_{0.12}$ | $1_0$ | $1_0$ |
| | 2 | $1664.24_{598.47}$ | $21720.48_{7192.31}$ | $26.44_{1.06}$ | $0.01_0$ | $0.5_{0.02}$ | $2.32_{0.04}$ | $1_0$ | $1_0$ |
| **usw** | 3 | $3041.01_{583.12}$ | $37929.45_{7130.12}$ | $24.96_{0.12}$ | $0_0$ | $0.54_{0.02}$ | $2.34_{0.01}$ | $1_0$ | $1_0$ |
| | 4 | $1482.3_{1295.18}$ | $60411.82_{37942.75}$ | $100.55_{25.33}$ | $0.19_{0.2}$ | $0.63_{0.08}$ | $1.88_{0.28}$ | $1_0$ | $1_0$ |
| | 5 | $2564.18_{1216.57}$ | $85908.57_{28969.87}$ | $72.58_{12.66}$ | $0.04_{0.03}$ | $0.59_{0.04}$ | $2.11_{0.11}$ | $1_0$ | $1_0$ |
| | 6 | $392.98_{358.23}$ | $30770.63_{31941.48}$ | $125.66_{49.46}$ | $0.46_{0.17}$ | $0.75_{0.08}$ | $1.53_{0.18}$ | $1_0$ | $1_0$ |
| | 7 | $724.31_{517.63}$ | $54555.59_{36874.79}$ | $150.15_{11.71}$ | $0.31_{0.17}$ | $0.69_{0.05}$ | $1.68_{0.18}$ | $1_0$ | $1_0$ |
| | 8 | $126.41_{38.3}$ | $5472.07_{3486.59}$ | $80.43_{23.21}$ | $0.65_{0.08}$ | $0.82_{0.04}$ | $1.34_{0.08}$ | $1_0$ | $1_0$ |
| | 9 | $137.14_{77.98}$ | $7707.84_{6948.17}$ | $94.31_{33.58}$ | $0.76_{0.12}$ | $0.86_{0.05}$ | $1.23_{0.12}$ | $1_0$ | $1_0$ |
| | 10 | $129.54_{17}$ | $5739_{1303.12}$ | $87.29_{10.69}$ | $0.68_{0.02}$ | $0.83_0$ | $1.31_{0.02}$ | $1_0$ | $1_0$ |
| | 2 | $675.06_{617.18}$ | $12372.51_{9253.91}$ | $41.61_{18.27}$ | $0.18_{0.21}$ | $0.58_{0.15}$ | $1.93_{0.32}$ | $1_0$ | $1_0$ |
| **bsc** | 3 | $674.4_{596.53}$ | $19009.72_{15837.79}$ | $58.82_{17.54}$ | $0.21_{0.2}$ | $0.63_{0.11}$ | $1.85_{0.28}$ | $1_0$ | $1_0$ |
| | 4 | $751.39_{1068.2}$ | $27887.16_{32217.37}$ | $77.93_{22.55}$ | $0.31_{0.25}$ | $0.68_{0.13}$ | $1.72_{0.31}$ | $1_0$ | $1_0$ |
| | 5 | $461.03_{705.52}$ | $20919.83_{29958.57}$ | $77.61_{27.9}$ | $0.46_{0.3}$ | $0.76_{0.13}$ | $1.54_{0.32}$ | $1_0$ | $1_0$ |
| | 6 | $240.02_{440.31}$ | $12698.73_{26078.34}$ | $83.92_{29.49}$ | $0.59_{0.23}$ | $0.8_{0.09}$ | $1.41_{0.23}$ | $1_0$ | $1_0$ |
| | 7 | $158.74_{144.52}$ | $8268.63_{11296.75}$ | $83.21_{31.8}$ | $0.65_{0.18}$ | $0.83_{0.07}$ | $1.34_{0.18}$ | $1_0$ | $1_0$ |
| | 8 | $117.99_{112.51}$ | $5651.66_{9206.02}$ | $73.82_{28.99}$ | $0.75_{0.15}$ | $0.87_{0.06}$ | $1.24_{0.15}$ | $1_0$ | $1_0$ |
| | 9 | $73.84_{43.2}$ | $2566.15_{3007.44}$ | $57.54_{22.32}$ | $0.84_{0.1}$ | $0.9_{0.04}$ | $1.15_{0.1}$ | $1_0$ | $1_0$ |
| | 10 | $59.45_{31.52}$ | $1766.42_{1773.52}$ | $49.11_{20.44}$ | $0.87_{0.08}$ | $0.92_{0.04}$ | $1.12_{0.14}$ | $1_0$ | $1_0$ |

**Table 2: General LON and basins' statistics for the JIB heuristic.**

| $\mathcal{T}$ | $C$ | $\overline{n_v}$ | $\overline{n_e}$ | $\overline{z}$ | $\overline{C_r}$ | $\overline{C}$ | $\overline{l}$ | $\pi$ | $\overline{S}$ |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | $387.14_{365.26}$ | $9183.38_{13001.81}$ | $39.09_{25.26}$ | $0.27_{0.28}$ | $0.55_{0.26}$ | $1.44_{0.91}$ | $0.9_{0.28}$ | $35.85_{131.14}$ |
| **unc** | 3 | $376.01_{446.51}$ | $12933.22_{22815.43}$ | $42.69_{33.54}$ | $0.36_{0.32}$ | $0.66_{0.22}$ | $2.92_{0.82}$ | $0.93_{0.25}$ | $6.47_{25.46}$ |
| | 4 | $320.67_{492.12}$ | $12570.34_{26488.7}$ | $52.26_{33.63}$ | $0.39_{0.28}$ | $0.73_{0.13}$ | $1.47_{0.68}$ | $0.95_{0.19}$ | $1.18_{1.24}$ |
| | 5 | $233.42_{243.77}$ | $9516.94_{14458.59}$ | $57.78_{33.89}$ | $0.39_{0.23}$ | $0.73_{0.14}$ | $1.47_{0.58}$ | $0.96_{0.19}$ | $1.37_{2.5}$ |
| | 6 | $194.99_{174.73}$ | $7490.62_{10059.34}$ | $55.48_{30.93}$ | $0.45_{0.26}$ | $0.78_{0.09}$ | $1.48_{0.44}$ | $0.98_{0.14}$ | $1.39_{3.8}$ |
| | 7 | $126.03_{135.36}$ | $4494.36_{7313.9}$ | $49.7_{26.9}$ | $0.57_{0.23}$ | $0.82_{0.08}$ | $1.42_{0.24}$ | $1_0$ | $1_0$ |
| | 8 | $74.71_{53.4}$ | $2042.5_{2535.87}$ | $41.24_{21.61}$ | $0.64_{0.19}$ | $0.84_{0.07}$ | $1.3_{0.38}$ | $0.98_{0.14}$ | $1.09_{0.8}$ |
| | 9 | $52.66_{34.52}$ | $1133.33_{1288.86}$ | $34.09_{15.65}$ | $0.75_{0.17}$ | $0.87_{0.08}$ | $1.25_{0.18}$ | $1_0$ | $1_0$ |
| | 10 | $35.89_{24.86}$ | $601.03_{754.05}$ | $26.27_{11.84}$ | $0.84_{0.15}$ | $0.92_{0.06}$ | $1.15_{0.15}$ | $1_0$ | $1_0$ |
| | 2 | $1323.35_{583.24}$ | $49482.86_{27283.31}$ | $68.47_{29.66}$ | $0.05_{0.02}$ | $0.23_{0.11}$ | $1.77_{0.98}$ | $0.88_{0.32}$ | $62.65_{171.6}$ |
| **usw** | 3 | $565.16_{352.61}$ | $20563.04_{17739.61}$ | $58.86_{34.2}$ | $0.11_{0.07}$ | $0.36_{0.17}$ | $1.37_{1.12}$ | $0.82_{0.38}$ | $45.47_{95.42}$ |
| | 4 | $1347.42_{1343.72}$ | $73476.11_{69681}$ | $99.21_{44.97}$ | $0.16_{0.13}$ | $0.59_{0.11}$ | $1.5_{0.99}$ | $0.87_{0.33}$ | $1.54_{1.78}$ |
| | 5 | $966.26_{1025.38}$ | $67640.3_{72485.31}$ | $116.73_{49.69}$ | $0.36_{0.29}$ | $0.66_{0.16}$ | $1.51_{0.78}$ | $1_0$ | $1_0$ |
| | 6 | $477.38_{379.32}$ | $31049.01_{37190.96}$ | $100.73_{46.12}$ | $0.24_{0.06}$ | $0.72_{0.06}$ | $1.76_{0.07}$ | $1_0$ | $1_0$ |
| | 7 | $603.95_{493.46}$ | $53514.32_{43997.71}$ | $142.09_{65.47}$ | $0.4_{0.24}$ | $0.71_{0.12}$ | $1.57_{0.35}$ | $0.99_{0.1}$ | $1.06_{0.6}$ |
| | 8 | $154.05_{33.9}$ | $5261.68_{2411.25}$ | $66.2_{14.5}$ | $0.44_{0.07}$ | $0.78_{0.02}$ | $1.55_{0.07}$ | $1_0$ | $1_0$ |
| | 9 | $133.16_{82.67}$ | $6758.7_{6925.32}$ | $76.72_{42.27}$ | $0.57_{0.12}$ | $0.82_{0.05}$ | $1.37_{0.36}$ | $0.98_{0.14}$ | $1.02_{0.14}$ |
| | 10 | $93.19_{10.58}$ | $2590.2_{773.89}$ | $54.36_{12.14}$ | $0.58_{0.08}$ | $0.79_{0.02}$ | $1.41_{0.08}$ | $1_0$ | $1_0$ |
| | 2 | $460.48_{545.19}$ | $15032.99_{18235.33}$ | $53.76_{29.55}$ | $0.27_{0.23}$ | $0.59_{0.19}$ | $1.77_{0.3}$ | $0.96_{0.17}$ | $1.47_{4.15}$ |
| **bsc** | 3 | $586.22_{676.15}$ | $26870.47_{34510.12}$ | $67.94_{40.08}$ | $0.31_{0.26}$ | $0.62_{0.19}$ | $1.55_{0.65}$ | $0.94_{0.22}$ | $7.27_{41.69}$ |
| | 4 | $580.64_{790.3}$ | $32339.18_{42829.18}$ | $86.15_{45.67}$ | $0.34_{0.24}$ | $0.66_{0.15}$ | $1.47_{0.72}$ | $0.93_{0.25}$ | $2.85_{11.65}$ |
| | 5 | $716.18_{927.8}$ | $41679.47_{55873.16}$ | $89.52_{40.42}$ | $0.37_{0.26}$ | $0.7_{0.14}$ | $1.61_{0.38}$ | $0.99_{0.1}$ | $1.02_{0.2}$ |
| | 6 | $399.69_{566.03}$ | $26041.55_{42342.31}$ | $89.87_{46.88}$ | $0.48_{0.26}$ | $0.76_{0.11}$ | $1.49_{0.36}$ | $0.99_{0.1}$ | $1.05_{0.5}$ |
| | 7 | $255.56_{298.19}$ | $14855.05_{21399.29}$ | $86.04_{40.42}$ | $0.53_{0.21}$ | $0.79_{0.08}$ | $1.46_{0.21}$ | $1_0$ | $1_0$ |
| | 8 | $160.66_{170.85}$ | $8491.19_{13020.19}$ | $74.02_{38.45}$ | $0.65_{0.22}$ | $0.83_{0.08}$ | $1.32_{0.32}$ | $0.99_{0.1}$ | $1.01_{0.1}$ |
| | 9 | $100.06_{70.22}$ | $3925.34_{4614.94}$ | $62.25_{26.43}$ | $0.72_{0.16}$ | $0.86_{0.06}$ | $1.27_{0.16}$ | $1_0$ | $1_0$ |
| | 10 | $77.76_{42.95}$ | $2547.98_{2400.65}$ | $55.1_{21.02}$ | $0.77_{0.13}$ | $0.87_{0.06}$ | $1.22_{0.21}$ | $1_0$ | $1_0$ |

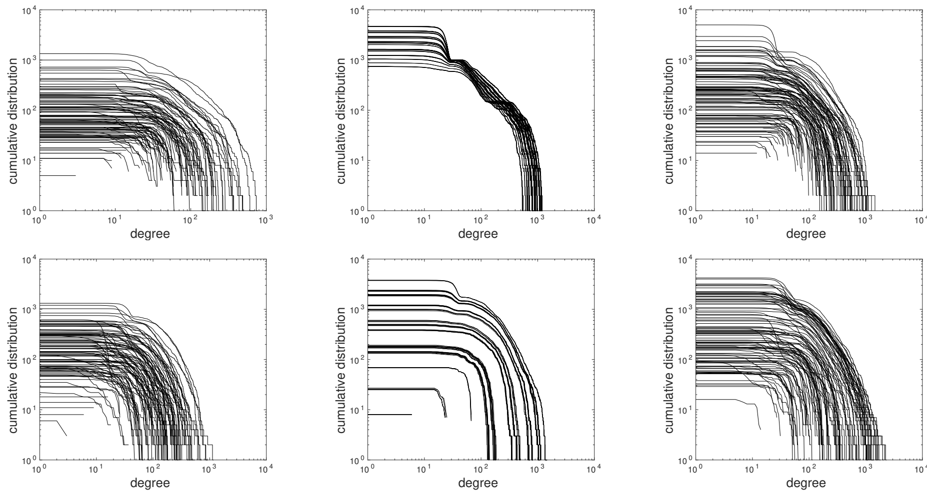



**Figure 2: Cumulative degree distribution of J2B (top) and JIB (bottom) for $\mathcal{T} = unc$, $C = 5$ (left), $\mathcal{T} = usw$, $C = 5$ (middle), and $\mathcal{T} = bsc$, $C = 5$ (right). All curves are shown in a log-log scale.**

**Table 3: The rates at which the Kolmogorov-Smirnov test fails to reject power-law and exponential as plausible distribution models, with a significance level of 0.1**

| | | T=unc, C=5 | T=usw, C=5 | T=bsc, C=5 |
|---|---|---|---|---|
| Power-law | J2B | 0.22 | 1 | 0.53 |
| | JIB | 0.39 | 0.26 | 0.46 |
| Exponential | J2B | 1 | 1 | 1 |
| | JIB | 1 | 1 | 1 |

degree distribution allows one to search a power-law graph more rapidly, relying on the fact that the number of edges per node varies considerably from node to node, i.e., its edges do not let us uniformly sample the graph, but they preferentially lead to high degree nodes. This again supports our conjecture about the existence of few nodes with high degree that efficiently connect the entire landscape. A local search algorithm has more chances to move to one of these high degree nodes than to other random nodes, which allows the algorithm to efficiently browse the entire graph.

With the aim of performing a rigorous study on the cumulative degree distributions, we use the Kolmogorov-Smirnov test to investigate the adequacy of power-law [4] and exponential models [6]. The test is performed on all 100 samples for each TTP class shown in Figure 2, and the rates at which the test fails to reject the null hypothesis (that the data come from the considered distribution model) are shown in Table 3.

Although J2B was able to produce LONs with degree distributions that fit a power-law distribution for many instances, a power-law can not be generalised as a plausible model to describe the degree distribution for all the landscape.

Given this fact, another possibility is to look into fitting the degree distribution to an exponential model of the form $P(k) = \frac{e^{-k/z}}{z}$. This model was proposed by [17] to describe the degree distributions for NK models, with tunable problem difficulties. Looking at Figure 2, especially the curves produced by JIB, an exponential model seems to be a good alternative. Table 3 shows that the Kolmogorov-Smirnov always fails to reject the exponential distribution as a plausible model for all the samples considered.

## 5.3 Basins of attraction

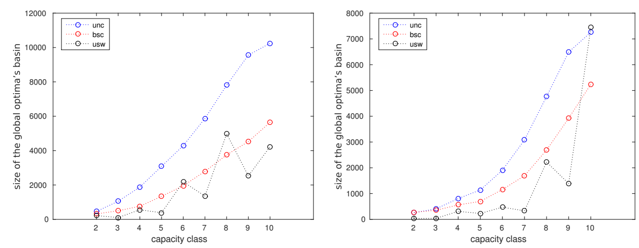



**Figure 3: Average of the relative size of the basin corresponding to the global maximum for each $C$ over the 100 TTP instances for J2B (left) and JIB (right).**

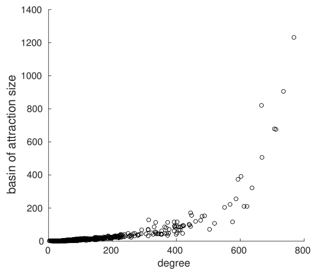
Exponential degree distributions do not provide a straightforward interpretation of the local search strategies' behaviour as power-law does. Therefore, an alternative approach to analyse the difficulty of the search space for the considered heuristics is to look into the size of the basins of attraction. In Figure 3 we illustrate the mean sizes of global optimum basins over all the instances for all the capacity classes. The plots show that the basin size generally increases when the capacity class increases.

This correlation is more thoroughly examined and generalized to the mean size of all basins of attraction for some representative instances as shown in Figure 4. We can clearly see a correlation between the degrees and the basin sizes.

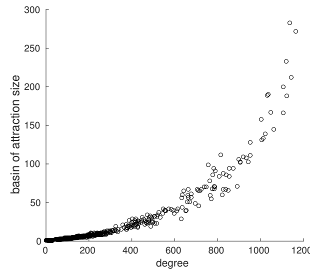
Next, we take a look into the relationship between the fitness of local optima and their basin size. Figure 5 illustrates this relationship for representative instances for the two heuristics.

For J2B, there is a clear positive correlation between the fitness of local optima and their basin size as the size of the basin increases when the fitness value increases. The fact that the solutions having the highest fitnesses belong to large basins makes them more likely to be found using the addressed local search techniques. Furthermore, we can also observe that the number of good quality local optima is smaller compared to low quality local optima.

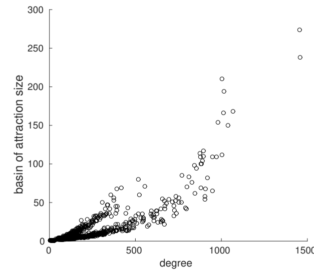
On the other hand, it is difficult to extract a pattern for JIB due to the high amount of volatility at which the fitnesses are scattered



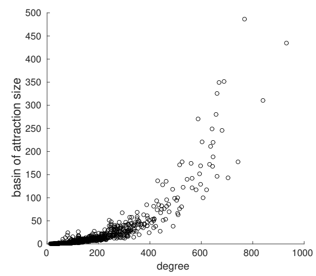

**(a) J2B for $\mathcal{T} = unc$ and $C = 5$.**



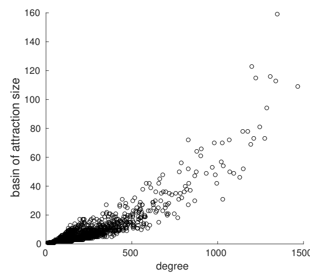

**(b) J2B for $\mathcal{T} = usw$ and $C = 5$.**



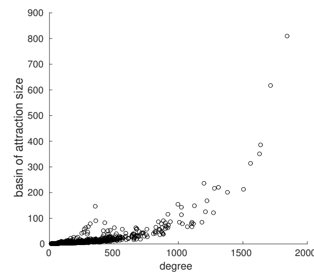

**(c) J2B for $\mathcal{T} = bsc$ and $C = 5$.**




**(d) JIB for $\mathcal{T} = unc$ and $C = 5$.**

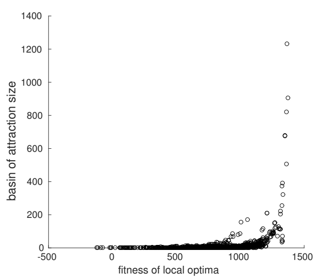



**(e) JIB for $\mathcal{T} = usw$ and $C = 5$.**

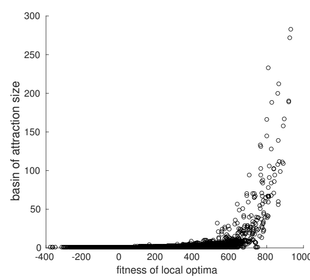



**(f) JIB for $\mathcal{T} = bsc$ and $C = 5$.**

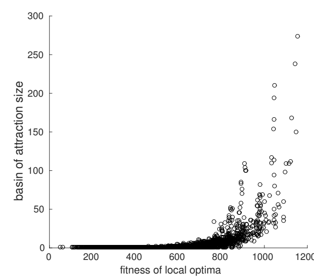
**Figure 4: Correlation between the degree of local optima and their corresponding basin sizes on representative examples.**



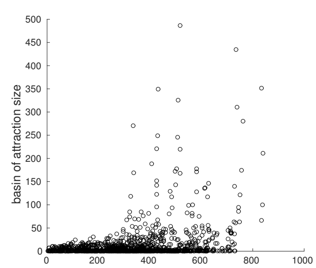

**(a) J2B for $\mathcal{T} = unc$ and $C = 5$.**



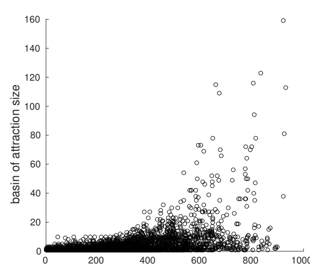

**(b) J2B for $\mathcal{T} = usw$ and $C = 5$.**



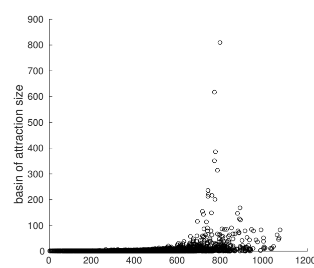

**(c) J2B for $\mathcal{T} = bsc$ and $C = 5$.**




**(d) JIB for $\mathcal{T} = unc$ and $C = 5$.**




**(e) JIB for $\mathcal{T} = usw$ and $C = 5$.**




**(f) JIB for $\mathcal{T} = bsc$ and $C = 5$.**

**Figure 5: Correlation between the fitness of local optima and their corresponding basin sizes on representative examples.**

relatively to the basin sizes. Therefore, it is safe to assume that the probability that JIB will identify a global optimum is quite low.

At first sight, it is not clear how this information can be used in a heuristic. However, if techniques like self-adaptation and restarts are used in combination with J2B, then the progress achieved over time can be used in online control as an indicator for the expected achievable solution quality.

Also, if the same local optimum is found over and over again from random starting points, we conjecture that such restarts are not necessary for J2B, whereas exploring these observations is quite difficult in the case of JIB.

From a comparison point of view, J2B seems to be a better local search approach compared to JIB. In a sense, this is not surprising as most well performing heuristics for the TTP, including memetic algorithms, apply the *2-OPT* and *bit-flip* operators in their algorithms mainly to improve their exploitation ability [7, 16, 25]. Note that we are not dismissing the *insertion* operator. In fact, the *insertion* move has been commonly adopted as a disruptive operator in state-of-the-art TSP solvers [15].

# 6 CONCLUSION

In this paper, the structure of the Local Optima Networks (LONs) and the basins of attraction in the Travelling Thief Problem (TTP) were investigated. Two problem features were studied, namely, the knapsack capacity, and the correlation between weights and item profits, in order to quantify their impact on the problem difficulty. We proposed a problem classification based on these two features and examined enumerable TTP instances using two basic local search heuristics: J2B, which combines the *2-OPT* and *bit-flip* operators, and JIB, which combines the *insertion* and *bit-flip* operators. For each instance, the corresponding LON was built aiming to extract some useful network measures.

Based on the reported results, we concluded that instances with high knapsack capacities might be easier to solve. The results also showed that the investigated LONs have two small-world properties: strong local clustering and a small mean path length. These properties suggest the existence of dense local connections between some local optima, and that almost any two local optima are connected with few local search iterations. Furthermore, some LONs built using JIB encompass disconnected sub-graphs, while the LONs generated using J2B are always connected, which in turn may make the instances easier to solve for J2B.

In addition to the LON topological properties, the cumulative degree distribution was analysed for a representative set of instances. The Kolmogorov-Smirnov test was used to investigate the adequacy of a power-law and an exponential model. The test showed that, differently from the exponential model, the power-law can not be generalised for all the studied LONs. However, many LONs showed a scale-free behaviour as the Kolmogorov-Smirnov test failed to reject the power-law as a plausible distribution.

The study of attraction basins confirmed that the difficulty of instances drops significantly when the knapsack capacity increases. Finally, the investigation of the relationship between the fitness of local optima and their basin size shows that J2B has better chances to identify the global optima compared to JIB.

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